PROOF OF FORMULA 3.546.1

$$\int_0^\infty e^{-bx^2} \sinh ax \, dx = \frac{\sqrt{\pi}}{2\sqrt{b}} \exp\left(\frac{a^2}{4b}\right) \operatorname{erf}\left(\frac{a}{2\sqrt{b}}\right)$$

Let $t = \sqrt{b}x$ to produce

$$\int_0^\infty e^{-bx^2} \sinh ax \, dx = \frac{1}{\sqrt{b}} J$$

with

$$J = \int_0^\infty e^{-t^2} \sinh ct \, dt$$

and $c = a/\sqrt{b}$. Then write

$$\begin{aligned} J &= \frac{1}{2} e^{c^2/4} \int_0^\infty e^{-(t-c/2)^2} \, dt - \frac{1}{2} e^{c^2/4} \int_0^\infty e^{-(t+c/2)^2} \, dt \\ &= \frac{1}{2} e^{c^2/4} \int_{-c/2}^\infty e^{-u^2} \, du - \frac{1}{2} e^{c^2/4} \int_{c/2}^\infty e^{-u^2} \, du \\ &= e^{c^2/4} \int_0^{c/2} e^{-u^2} \, du. \end{aligned}$$

The previous expression, when written in terms of the error function defined by

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-u^2} \, du,$$

gives the result.