PROOF OF FORMULA 3.551.4

$$\int_0^\infty x^n e^{-(p+mq)x} \sinh^m qx\, dx = \frac{n!}{2^m} \sum_{k=0}^{m} \binom{m}{k} \frac{(-1)^k}{(p+2kq)^{n+1}}$$

Write the integral as

$$\begin{aligned} \int_0^\infty x^n e^{-(p+mq)x} \sinh^m qx\, dx &= 2^{-m} \int_0^\infty x^n e^{-(p+mq)x} (e^{qx} - e{-}qx)^m\, dx \\ &= 2^{-m} \sum_{k=0}^{m} (-1)^k \binom{m}{k} \int_0^\infty x^n e^{-(p+2kq)x}\, dx. \end{aligned}$$

The change of variables $t = (p+2kq)x$ gives the result.