PROOF OF FORMULA 3.551.7

$$\int_1^\infty \frac{e^{-bx}}{x} \cosh ax \, dx = -\frac{1}{2}\left[\text{Ei}(a-b) + \text{Ei}(-a-b)\right]$$

The integral is

$$\begin{aligned}\int_1^\infty \frac{e^{-bx}}{x} \cosh ax \, dx &= \frac{1}{2}\int_1^\infty \frac{e^{-(b-a)x}}{x} dx + \frac{1}{2}\int_1^\infty \frac{e^{-(b+a)x}}{x} dx \\ &= \frac{1}{2}\int_{b-a}^\infty \frac{e^{-t}}{t} dt + \frac{1}{2}\int_{b+a}^\infty \frac{e^{-t}}{t} dt.\end{aligned}$$

The definition

$$\text{Ei}(-z) := -\int_{-z}^\infty \frac{e^{-t}}{t} dt$$

gives the result.