

PROOF OF FORMULA 3.553.2

$$\int_0^{\infty} \frac{\sinh^2(x/2)}{\cosh x} \frac{e^{-x}}{x} dx = \frac{1}{2} \ln \left(\frac{4}{\pi} \right)$$

The change of variables $t = e^{-x}$ gives

$$\int_0^{\infty} \frac{\sinh^2(x/2)}{\cosh x} \frac{e^{-x}}{x} dx = -\frac{1}{2} \int_0^1 \frac{(1-t)^2}{1+t^2} \frac{dt}{\ln t}.$$

This integral has value $\ln \pi/4$ and it appears as entry 4.267.2.