

**PROOF OF FORMULA 3.562.4**

$$\int_0^{\infty} x e^{-\beta x^2} \cosh \gamma x \, dx = \frac{\gamma}{4\beta} \sqrt{\frac{\pi}{\beta}} e^{\gamma^2/4\beta} \operatorname{erf} \left( \frac{\gamma}{2\sqrt{\beta}} \right) + \frac{1}{2\beta}$$

Write the integral as

$$\begin{aligned} \int_0^{\infty} x e^{-\beta x^2} \cosh \gamma x \, dx &= \frac{1}{2} \int_0^{\infty} x e^{-\beta x^2 + \gamma x} \, dx + \frac{1}{2} \int_0^{\infty} x e^{-\beta x^2 - \gamma x} \, dx \\ &+ \frac{1}{2} e^{\gamma^2/4\beta} \int_0^{\infty} x e^{-\beta(x-\gamma/2\beta)^2} \, dx + \frac{1}{2} e^{\gamma^2/4\beta} \int_0^{\infty} x e^{-\beta(x+\gamma/2\beta)^2} \, dx \\ &+ \frac{1}{2} e^{\gamma^2/4\beta} \int_{-\gamma/2\beta}^{\infty} (t + \gamma/2\beta) e^{-\beta t^2} \, dt + \frac{1}{2} e^{\gamma^2/4\beta} \int_{\gamma/2\beta}^{\infty} (t - \gamma/2\beta) e^{-\beta t^2} \, dt. \end{aligned}$$

Now let  $s = \sqrt{\beta}t$  to obtain

$$\begin{aligned} \int_0^{\infty} x e^{-\beta x^2} \cosh \gamma x \, dx &= \frac{1}{4\beta} e^{\gamma^2/4\beta} \int_{-\gamma/2\sqrt{\beta}}^{\infty} 2s e^{-s^2} \, ds + \frac{\gamma}{4\beta\sqrt{\beta}} e^{\gamma^2/4\beta} \int_{-\gamma/2\sqrt{\beta}}^{\infty} e^{-s^2} \, ds \\ &+ \frac{1}{4\beta} e^{\gamma^2/4\beta} \int_{\gamma/2\sqrt{\beta}}^{\infty} 2s e^{-s^2} \, ds - \frac{\gamma}{4\beta\sqrt{\beta}} e^{\gamma^2/4\beta} \int_{\gamma/2\sqrt{\beta}}^{\infty} e^{-s^2} \, ds \\ &= \frac{1}{2\beta} + \frac{\gamma e^{\gamma^2/4\beta}}{4\beta\sqrt{\beta}} \frac{\sqrt{\pi}}{2} \left[ \operatorname{erf} \left( \frac{\gamma}{2\sqrt{\beta}} \right) + \frac{1}{2} \right] \\ &\quad - \frac{\gamma e^{\gamma^2/4\beta}}{4\beta\sqrt{\beta}} \frac{\sqrt{\pi}}{2} \left[ -\operatorname{erf} \left( \frac{\gamma}{2\sqrt{\beta}} \right) + \frac{1}{2} \right] \end{aligned}$$

and this reduces to the stated result.