

**PROOF OF FORMULA 3.611.1**

$$\int_0^{2\pi} (1 - \cos x)^n \sin nx \, dx = 0$$

Define

$$I_m = \int_0^\pi \sin^m t \sin mt \, dt$$

and integrate by parts to obtain

$$I_m = \int_0^\pi \sin^{m-1} t \cos mt \cos t \, dt.$$

The identity  $\cos(m-1)t = \cos mt \cos t + \sin mt \sin t$  yields

$$2I_m = J_{m-1} := \int_0^\pi \sin^{m-1} t \cos(m-1)t \, dt.$$

A similar argument yields  $2J_m = -I_{m-1}$ . Therefore

$$I_m = -\frac{1}{4}I_{m-2} \text{ and } J_m = -\frac{1}{4}J_{m-2}.$$

The recurrence shows that  $I_{2n} = (-1)^n I_0 / 2^{2n}$ . The value  $I_0 = 0$  gives  $I_{2n} = 0$ .

To evaluate the integral presented here, write  $1 - \cos x = 2 \sin^2(x/2)$  to produce

$$\int_0^{2\pi} (1 - \cos x)^n \sin nx \, dx = 2^{n+1} \int_0^\pi \sin^{2n} t \sin 2nt \, dt = 2^{n+1} I_{2n} = 0,$$

as claimed.