

**PROOF OF FORMULA 3.612.2**

$$\int_0^\pi \frac{\sin nx}{\sin x} dx = \begin{cases} 0 & \text{if } n \text{ is even} \\ \pi & \text{if } n \text{ is odd} \end{cases} \quad n \geq 0$$

*Proof.* Assume  $n \geq 2$  and use

$$\sin nx = \sin(n-1)x \cos x + \cos(n-1)x \sin x$$

to obtain

$$\int_0^\pi \frac{\sin nx}{\sin x} dx = \int_0^\pi \frac{\sin(n-1)x \cos x}{\sin x} dx + \int_0^\pi \cos(n-1)x dx.$$

The last integral vanishes. Now use

$$\sin(n-1)x \cos x = \frac{1}{2} (\sin nx + \sin(n-2)x)$$

to obtain

$$\int_0^\pi \frac{\sin nx}{\sin x} dx = \int_0^\pi \frac{\sin(n-2)x}{\sin x} dx.$$

The result now follows by induction. □