PROOF OF FORMULA 3.621.1

$$\int_0^{\pi/2} \sin^{\mu-1} x \, dx = \int_0^{\pi/2} \cos^{\mu-1} x \, dx = 2^{\mu-2} B\left(\frac{\mu}{2}, \frac{\mu}{2}\right)$$

In the integral representation

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1} \, dt,$$

let $t = \sin^2 x$ to obtain

$$B(a,b) = 2 \int_0^{\pi/2} \sin^{2a-1} x \, \cos^{2b-1} x \, dx.$$

Now let $a = \mu/2$ and $b = 1/2$ to obtain

$$\int_0^{\pi/2} \sin^{\mu-1} x \, dx = \frac{1}{2} B\left(\frac{\mu}{2}, \frac{1}{2}\right).$$

The duplication formula of the gamma function

$$2^{2z-1}\Gamma(z)\Gamma(z+\tfrac{1}{2}) = \sqrt{\pi}\,\Gamma(2z),$$

can be written as

$$B\left(z, \tfrac{1}{2}\right) = 2^{2z-1} B(z,z).$$

This gives the stated form.