PROOF OF FORMULA 3.621.2

$$\int_0^{\pi/2} \sin^{3/2} x \, dx = \int_0^{\pi/2} \cos^{3/2} x \, dx = \frac{1}{6\sqrt{2\pi}} \Gamma^2 \left(\tfrac{1}{4} \right)$$

In the integral representation

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1} \, dt,$$

let $t = \sin^2 x$ to obtain

$$B(a,b) = 2 \int_0^{\pi/2} \sin^{2a-1} x \, \cos^{2b-1} x \, dx.$$

Now let $a = \mu/2$ and $b = 1/2$ to obtain

$$\int_0^{\pi/2} \sin^{\mu-1} x \, dx = \frac{1}{2} B \left(\frac{\mu}{2}, \frac{1}{2} \right).$$

The special case $\mu = \frac{5}{2}$ gives

$$\int_0^{\pi/2} \sin^{3/2} x \, dx = \frac{1}{2} B \left(\tfrac{5}{4}, \tfrac{1}{2} \right) = \frac{\Gamma(\frac{5}{4})\Gamma(\frac{1}{2})}{2\Gamma(\frac{7}{4})}.$$

This is simplified using

$$\Gamma(a+1) = a\Gamma(a) \text{ and } \Gamma(a)\Gamma(1-a) = \pi / \sin \pi a.$$