# PROOF OF FORMULA 3.621.3

$$\int_0^{\pi/2} \sin^{2m} x\,dx = \int_0^{\pi/2} \cos^{2m} x\,dx = \frac{(2m-1)!!}{(2m)!!}\frac{\pi}{2}$$

Let $t = \sin^x$ to obtain

$$\int_0^{\pi/2} \sin^{2m} x\,dx = \int_0^1 t^{2m}(1-t^2)^{-1/2}\,dt$$

The change of variables $s = t^2$ gives

$$\int_0^{\pi/2} \sin^{2m} x\,dx = \frac{1}{2}\int_0^1 s^{m-1/2}(1-s)^{-1/2}\,ds = \frac{1}{2}B\left(m+\tfrac{1}{2},\tfrac{1}{2}\right).$$

Therefore

$$\int_0^{\pi/2} \sin^{2m} x\,dx = \frac{\Gamma(m+1/2)\Gamma(1/2)}{2\Gamma(m+1)}.$$

The result now follows from

$$\Gamma(m+\tfrac{1}{2}) = \frac{\sqrt{\pi}}{2^m}(2m-1)!!$$