

**PROOF OF FORMULA 3.621.4**

$$\int_0^{\pi/2} \sin^{2m+1} x \, dx = \int_0^{\pi/2} \cos^{2m+1} x \, dx = \frac{(2m)!!}{(2m+1)!!}$$

Let  $t = \sin x$  to obtain

$$\int_0^{\pi/2} \sin^{2m+1} x \, dx = \int_0^1 t^{2m+1} (1-t^2)^{-1/2} dt$$

The change of variables  $s = t^2$  gives

$$\int_0^{\pi/2} \sin^{2m+1} x \, dx = \frac{1}{2} \int_0^1 s^m (1-s)^{-1/2} ds = \frac{1}{2} B\left(m+1, \frac{1}{2}\right).$$

Therefore

$$\int_0^{\pi/2} \sin^{2m+1} x \, dx = \frac{\Gamma(m+1)\Gamma(1/2)}{2\Gamma(m+3/2)}.$$

The result now follows from

$$\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^m} (2m-1)!!$$