

PROOF OF FORMULA 3.624.4

$$\int_0^{\pi/4} \frac{\cos^\mu(2x)}{(\cos x)^{2(\mu+1)}} dx = 2^{2\mu} B(\mu + 1, \mu + 1)$$

The integral can be written as

$$\int_0^{\pi/4} \frac{\cos^\mu(2x)}{(\cos x)^{2(\mu+1)}} dx = 2^{\mu+1} \int_0^{\pi/4} \frac{(\cos 2x)^\mu dx}{(1 + \cos 2x)^{\mu+1}}.$$

The change of variables $t = \cos 2x$ gives

$$\int_0^{\pi/4} \frac{\cos^\mu(2x)}{(\cos x)^{2(\mu+1)}} dx = 2^\mu \int_0^1 t^\mu (1-t)^{-1/2} (1+t)^{-\mu-3/2} dt.$$

An integral representation of the hypergeometric function is

$${}_2F_1[\alpha, \beta; \gamma; z] = \frac{1}{B(\beta, \gamma - \beta)} \int_0^1 t^{\beta-1} (1-t)^{\gamma-\beta-1} (1-tz)^{-\alpha} dt$$

shows that the original integral is

$$\int_0^{\pi/4} \frac{\cos^\mu(2x)}{(\cos x)^{2(\mu+1)}} dx = 2^\mu B(\mu + 1, \frac{1}{2}) {}_2F_1\left[\mu + \frac{3}{2}, \mu + 1; \mu + \frac{3}{2}; -1\right].$$

The result now follows from the specialization

$${}_2F_1[\alpha, \beta; \alpha; z] = (1-z)^{-\beta}.$$