PROOF OF FORMULA 3.624.5

$$\int_0^{\pi/4} \frac{\sin^{2\mu-2} x}{(\cos x)^{2\mu}} \, dx = 2^{1-2\mu} B(2\mu-1, 1-\mu)$$

Use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ and the change of variable $t = \cos 2x$ to obtain

$$\int_0^{\pi/4} \frac{\sin^{2\mu-2} x}{(\cos x)^{2\mu}} \, dx = \frac{1}{2^\mu} \int_0^1 t^{-\mu}(1-t)^{\mu-3/2}(1+t)^{-1/2} \, dt.$$

The integral representation

$${}_2F_1 \left[\alpha, \beta; \gamma; z\right] = \frac{1}{B(\beta, \gamma-\beta)} \int_0^1 t^{\beta-1}(1-t)^{\gamma-1}(1-tz)^{-\alpha} \, dt$$

gives the value

$$\begin{aligned} \int_0^{\pi/4} \frac{\sin^{2\mu-2} x}{(\cos x)^{2\mu}} \, dx &= \frac{1}{2^\mu} B(1-\mu, \mu-\tfrac{1}{2}) {}_2F_1 \left[\tfrac{1}{2}, 1-\mu; \tfrac{1}{2}; -1\right] \\ &= \frac{1}{2^\mu} B(1-\mu, \mu-\tfrac{1}{2}) 2^{\mu-1}. \end{aligned}$$

This is the result using

$$\Gamma(x+\tfrac{1}{2}) = \frac{\sqrt{\pi}\,\Gamma(2x)}{\Gamma(x)\, 2^{2x-1}}.$$