

PROOF OF FORMULA 3.638.1

$$\int_0^{\pi/4} \frac{\sin^{2\mu} x \, dx}{(\cos 2x)^{\mu+1/2} \cos x} = \frac{\pi}{2} \sec \pi \mu$$

The change of variables $t = \tan x$ gives

$$\int_0^{\pi/4} \frac{\sin^{2\mu} x \, dx}{(\cos 2x)^{\mu+1/2} \cos x} = \int_0^1 t^{2\mu} (1-t^2)^{-\mu-1/2} dt.$$

Now let $t = v^2$ to obtain

$$\int_0^{\pi/4} \frac{\sin^{2\mu} x \, dx}{(\cos 2x)^{\mu+1/2} \cos x} = \frac{1}{2} \int_0^1 v^{\mu-1/2} (1-v)^{-\mu-1/2} dv.$$

This last integral is

$$B\left(\mu + \frac{1}{2}, -\mu + \frac{1}{2}\right) = \Gamma\left(\mu + \frac{1}{2}\right)\Gamma\left(\frac{1}{2} - \mu\right) = \frac{\pi}{\cos \pi \mu}$$

and the result follows.