

PROOF OF FORMULA 3.638.2

$$\int_0^{\pi/4} \frac{\sin^{\mu-1/2} 2x \, dx}{\cos^\mu 2x \cos x} = \frac{2}{\sqrt{\pi}(2\mu-1)} \Gamma(\mu + \frac{1}{2}) \Gamma(1-\mu) \sin\left(\frac{(2\mu-1)\pi}{4}\right)$$

The change of variables $t = \tan x$ yields

$$\int_0^{\pi/4} \frac{\sin^{\mu-1/2} 2x \, dx}{\cos^\mu 2x \cos x} = 2^{\mu-1/2} \int_0^1 t^{\mu-1/2} (1-t^2)^{-\mu} dt.$$

Now let $v = t^2$ to obtain

$$\int_0^{\pi/4} \frac{\sin^{\mu-1/2} 2x \, dx}{\cos^\mu 2x \cos x} = 2^{\mu-3/2} \int_0^1 v^{\mu/2-3/4} (1-v)^{-\mu} dv.$$

This last integral is

$$B\left(\frac{\mu}{2} + \frac{1}{4}, 1-\mu\right) = \frac{\Gamma\left(\frac{\mu}{2} + \frac{1}{4}\right) \Gamma(1-\mu)}{\Gamma\left(\frac{5}{4} - \frac{\mu}{4}\right)}.$$

The result is obtained by using

$$\Gamma\left(x + \frac{1}{2}\right) = \frac{\Gamma(2x) \sqrt{\pi}}{2^{2x-1} \Gamma(x)}$$

to write

$$\Gamma\left(\frac{1}{4} + \frac{\mu}{2}\right) = \frac{\Gamma\left(\mu - \frac{1}{2}\right) \sqrt{\pi}}{2^{\mu-\frac{1}{2}} \Gamma\left(\frac{\mu}{2} - \frac{1}{4}\right)}.$$