

**PROOF OF FORMULA 3.644.3**

$$\int_0^\pi \frac{\sin^m x \, dx}{1 - \cos x} = 2^{m-1} B\left(\frac{m-1}{2}, \frac{m+1}{2}\right)$$

Write the integral as

$$\int_0^\pi \frac{\sin^m x \, dx}{1 - \cos x} = \int_0^\pi (1 + \cos x)^{m/2} (1 - \cos x)^{m/2-1} \, dx.$$

The change of variables  $t = 1 - \cos x$  produces

$$\begin{aligned} \int_0^\pi \frac{\sin^m x \, dx}{1 - \cos x} &= \int_0^\pi (1 + \cos x)^{m/2} (1 - \cos x)^{m/2-1} \, dx \\ &= \int_0^2 t^{m/2-1} (2-t)^{m/2} \frac{dt}{\sqrt{t(2-t)}} \\ &= 2^{m-1} \int_0^1 s^{m/2-3/2} (1-s)^{m/2-1/2} \, ds \end{aligned}$$

using  $t = 2s$  in the last step. This is the result.