PROOF OF FORMULA 3.662.1

$$\int_0^{\pi/2} (\sec x - 1)^{\mu} \sin x \, dx = \int_0^{\pi/2} (\operatorname{cosec} x - 1)^{\mu} \cos x \, dx = \frac{\pi\mu}{\sin \pi\mu}$$

The proof is given for the first integral. The second one is reduced to the first one by $x \mapsto \pi/2 - x$.

Let $t = \cos x$ to obtain

$$\int_0^{\pi/2} (\sec x - 1)^{\mu} \sin x \, dx = \int_0^1 t^{-\mu}(1-t)^{\mu} \, dt.$$

This last integral is

$$B(1-\mu, 1+\mu) = \Gamma(1-\mu, 1+\mu) = \mu\Gamma(\mu)\Gamma(1-\mu),$$

and this reduces to the stated answer.