

PROOF OF FORMULA 3.662.2

$$\int_0^{\pi/2} (\operatorname{cosec} x - 1)^\mu \sin 2x \, dx = \frac{(1 - \mu)\pi\mu}{\sin \pi\mu}$$

Write

$$\int_0^{\pi/2} (\operatorname{cosec} x - 1)^\mu \sin 2x \, dx = 2 \int_0^{\pi/2} (1 - \cos x)^\mu \sin x \cos^{1-\mu} x \, dx.$$

The change of variable $t = \cos x$ gives

$$\int_0^{\pi/2} (\operatorname{cosec} x - 1)^\mu \sin 2x \, dx = 2 \int_0^1 (1 - t)^\mu t^{1-\mu} \, dt.$$

This last integral is

$$2B(1 + \mu, 2 - \mu) = \frac{2\Gamma(1 + \mu)\Gamma(2 - \mu)}{\Gamma(3)},$$

and this simplifies to the stated answer.