PROOF OF FORMULA 4.212.3

$$\int_0^1 \frac{dx}{(a+\ln x)^2} = -\frac{1}{a} + e^{-a}\mathrm{Ei}(a)$$

Let $t = a + \ln x$ to obtain

$$\int_0^1 \frac{dx}{(a+\ln x)^2} = e^{-a}\int_{-\infty}^{a} \frac{e^t\,dt}{t^2}.$$

Now observe that

$$\frac{d}{dt}\left(\frac{e^t}{t}\right) = -\frac{e^t}{t^2} + \frac{e^t}{t},$$

and integrating from $-\infty$ to a yields

$$\frac{e^a}{a} = -\int_{-\infty}^{a} \frac{e^t\,dt}{t^2} + \int_{-\infty}^{a} \frac{e^t\,dt}{t}.$$

This gives

$$e^{-a}\int_{-\infty}^{a} \frac{e^t\,dt}{t^2} = -\frac{1}{a} + e^{-a}\int_{-\infty}^{a} \frac{e^t\,dt}{t},$$

and the result follows.