PROOF OF FORMULA 4.212.7

$$\int_1^e \frac{\ln x\, dx}{(1+\ln x)^2} = \frac{e}{2} - 1$$

Let $t = 1 + \ln x$ to obtain

$$\int_1^e \frac{\ln x\, dx}{(1+\ln x)^2} = \frac{1}{e}\int_1^2 \frac{t-1}{t^2} e^t\, dt.$$

Now observe that

$$\frac{t-1}{t^2} e^t\, dt = \frac{d}{dt}\left(\frac{e^t}{t}\right),$$

to obtain the result.