

**PROOF OF FORMULA 4.212.8**

$$\int_0^1 \frac{dx}{(a + \ln x)^n} = \frac{e^{-a} \text{Ei}(a)}{(n-1)!} - \frac{1}{(n-1)!} \sum_{k=1}^{n-1} \frac{(n-k-1)!}{a^{n-k}}$$

Let  $t = a + \ln x$  to obtain

$$\int_0^1 \frac{dx}{(a + \ln x)^n} = e^{-a} \int_{-\infty}^a \frac{e^t}{t^n} dt.$$

The indefinite integral is evaluated in 2.324.2 as

$$\int \frac{e^t}{t^n} dt = -\frac{e^t}{(n-1)!} \sum_{k=1}^{n-1} \frac{(n-k-1)!}{x^{n-k}} + \frac{\text{Ei}(t)}{(n-1)!}.$$

Now evaluate at  $t = -\infty$  and  $t = a$ . Keep in mind that  $\text{Ei}(-\infty) = 0$ .