

PROOF OF FORMULA 4.215.3

$$\int_0^1 \sqrt{\ln \frac{1}{x}} dx = \frac{\sqrt{\pi}}{2}$$

Let $t = \ln \frac{1}{x}$ to produce

$$\int_0^1 \left(\ln \frac{1}{x}\right)^{\mu-1} dx = \int_0^\infty t^{\mu-1} e^{-t} dt.$$

This is the standard integral representation of the gamma function, thus

$$\int_0^1 \left(\ln \frac{1}{x}\right)^{\mu-1} dx = \Gamma(\mu).$$

Now put $\mu = \frac{3}{2}$ to obtain the result.