FORMULA 4.222.2

$$\int_0^\infty \ln x \, \ln \frac{a^2 + x^2}{b^2 + x^2} \, dx = \pi \ln \frac{a^a}{b^b} - (a - b)\pi$$