PROOF OF FORMULA 4.223.3

$$\int_0^\infty \ln(1 + 2e^{-x}\cos t + e^{-2x})\,dx = \frac{\pi^2}{6} - \frac{t^2}{2}$$

The change of variables $y = e^{-x}$ gives

$$\int_0^\infty \ln(1 + 2e^{-x}\cos t + e^{-2x})\,dx = \int_0^1 \ln(1 + 2y\cos t + y^2)\,\frac{dy}{y}.$$

This integral appears as entry 4.296.1 and it gives the result.