PROOF OF FORMULA 4.224.12

$$\int_0^\pi \ln(1 + b\cos x)\,dx = \pi \ln\left(\frac{1+\sqrt{1-b^2}}{2}\right)$$

Entry 4.224.9 states that

$$\int_0^\pi \ln(a + b\cos x)\,dx = \pi \ln\left(\frac{a+\sqrt{a^2-b^2}}{2}\right).$$

This is the special case $a = 1$.