

**PROOF OF FORMULA 4.227.12**

$$\int_0^{\pi/2} \ln [(1 - \tan x)^2] dx = \frac{\pi}{2} \ln 2 - 2G$$

The integral is

$$\int_0^{\pi/2} \ln [(1 - \tan x)^2] dx = 2 \int_0^{\pi/4} \ln(1 - \tan x) dx + 2 \int_{\pi/4}^{\pi/2} \ln(\tan x - 1) dx.$$

The change of variables  $t = \pi/2 - x$  in the second integral yields

$$\int_0^{\pi/2} \ln [(1 - \tan x)^2] dx = 4 \int_0^{\pi/4} \ln(\cos x - \sin x) dx - 2 \int_0^{\pi/4} \ln \cos x dx - 2 \int_0^{\pi/4} \ln \sin x dx.$$

The result now follows from entries 4.225.1, 4.224.5 and 4.224.2 that give

$$\int_0^{\pi/4} \ln(\cos x - \sin x) dx = -\frac{\pi}{8} \ln 2 - \frac{G}{2},$$

$$\int_0^{\pi/4} \ln \cos x dx = -\frac{\pi}{4} \ln 2 + \frac{G}{2},$$

$$\int_0^{\pi/4} \ln \sin x dx = -\frac{\pi}{4} \ln 2 - \frac{G}{2},$$

respectively.