PROOF OF FORMULA 4.227.17

$$\int_0^{\pi/2} \ln\left(a^2 + b^2 \tan^2 x\right)\, dx = \frac{1}{2}\int_0^{\pi} \ln\left(a^2 + b^2 \tan^2 x\right)\, dx = \pi \ln(a+b)$$

Write the integral as

$$\int_0^{\pi/2} \ln\left(a^2 + b^2 \tan^2 x\right)\, dx = \int_0^{\pi/2} \ln\left(a^2 \cos^2 x + b^2 \sin^2 x\right)\, dx - 2\int_0^{\pi/2} \ln\cos x\, dx.$$

The result now follows from entry 4.226.6

$$\int_0^{\pi/2} \ln\left(a^2 \cos^2 x + b^2 \sin^2 x\right)\, dx = \pi \ln\left(\frac{a+b}{2}\right)$$

and 4.224.6

$$\int_0^{\pi/2} \ln\cos x\, dx = -\frac{\pi}{2}\ln 2.$$