PROOF OF FORMULA 4.227.2

$$\int_0^{\pi/4} \ln\tan x\,dx = -\int_{\pi/4}^{\pi/2} \ln\tan x\,dx = -G$$

The Catalan constant has the integral representation

$$G = -\int_0^1 \frac{\ln x\,dx}{1+x^2}.$$

This appears as 4.231.12. The change of variables $x = \tan t$ gives the first evaluation. The change of variables $x \mapsto \pi/2 - x$ shows that

$$\int_0^{\pi/2} \ln\tan x\,dx = 0.$$

This gives the second evaluation.