PROOF OF FORMULA 4.227.3

$$\int_0^{\pi/2} \ln(a \tan x)\, dx = \frac{\pi}{2} \ln a$$

Expanding the logarithm

$$\int_0^{\pi/2} \ln(a \tan x)\, dx = \frac{\pi}{2} \ln a + \int_0^{\pi/2} \ln(\tan x)\, dx.$$

The change of variables $t = \frac{\pi}{2} - x$ gives

$$\int_0^{\pi/2} \ln(\tan x)\, dx = \int_0^{\pi/2} \ln(\cot x)\, dx = -\int_0^{\pi/2} \ln(\tan x)\, dx.$$

This completes the proof.