

FORMULA 4.227.5

$$\begin{aligned}\int_0^{\pi/2} \ln^{2n} \tan x \, dx &= 2(2n)! \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^{2n+1}} \\ &= \left(\frac{\pi}{2}\right)^{2n+1} |E_{2n}| \text{ if } n \text{ is even}\end{aligned}$$

Observe that

$$\int_{\pi/4}^{\pi/2} \ln^{2n} \tan x \, dx = \int_0^{\pi/4} \ln^{2n} \cot x \, dx = \int_0^{\pi/4} \ln^{2n} \tan x \, dx$$

and now the result follows from entry 4.227.4.