PROOF OF FORMULA 4.227.9

$$\int_0^{\pi/4} \ln(1+\tan x)\,dx = \frac{\pi}{8}\ln 2$$

Write the integral as

$$\int_0^{\pi/4} \ln(1+\tan x)\,dx = \int_0^{\pi/4} \ln(\cos x+\sin x)\,dx - \int_0^{\pi/4} \ln\cos x\,dx.$$

The result now follows from the evaluations

$$\int_0^{\pi/4} \ln(\cos x+\sin x)\,dx = -\frac{\pi}{8}\ln 2 + \frac{G}{2}$$

and

$$\int_0^{\pi/4} \ln\cos x\,dx = -\frac{\pi}{4}\ln 2 + \frac{G}{2}$$

given in entries 4.225.2 and 4.224.5, respectively.