

PROOF OF FORMULA 4.229.3

$$\int_0^1 \ln(\ln 1/x) \frac{dx}{\sqrt{\ln 1/x}} = -(\gamma + 2 \ln 2) \sqrt{\pi}$$

Entry 4.229.4 states that

$$\int_0^1 \ln(\ln 1/x) (\ln 1/x)^{\mu-1} dx = \psi(\mu) \Gamma(\mu)$$

The result now follows from the values $\psi(1/2) = -(\gamma + 2 \ln 2)$ (given as entry 8.366.2) and $\Gamma(1/2) = \sqrt{\pi}$.