PROOF OF FORMULA 4.231.1

$$\int_0^1 \frac{\ln x\, dx}{1+x} = -\frac{\pi^2}{12}$$

Let $t = -\ln x$ to obtain

$$\int_0^1 \frac{\ln x\, dx}{1+x} = -\int_0^\infty \frac{t\, dt}{1+e^t}.$$

The integral representation 9.513.1 of the Riemann zeta function

$$\zeta(s) = \frac{1}{(1-2^{1-s})\Gamma(s)} \int_0^\infty \frac{t^{s-1}\, dt}{1+e^t}$$

now gives

$$\int_0^1 \frac{\ln x\, dx}{1+x} = -\frac{\zeta(2)}{2} = -\frac{\pi^2}{12}.$$