

PROOF OF FORMULA 4.231.11

$$\int_0^a \frac{\ln x \, dx}{x^2 + a^2} = \frac{\pi \ln a}{4a} - \frac{G}{a}$$

Let $x = at$ to obtain

$$\int_0^a \frac{\ln x \, dx}{x^2 + a^2} = \frac{\ln a}{a} \int_0^1 \frac{dt}{1+t^2} + \frac{1}{a} \int_0^1 \frac{\ln t \, dt}{1+t^2}.$$

The first integral is $\pi/4$ and the second one is $-G$ as computed in 4.231.12.