

PROOF OF FORMULA 4.231.13

$$\int_0^1 \frac{\ln x \, dx}{1-x^2} = -\frac{\pi^2}{8}$$

The partial fraction decomposition

$$\frac{1}{1-x^2} = \frac{1}{2} \left(\frac{1}{1+x} + \frac{1}{1-x} \right)$$

gives

$$\int_0^1 \frac{\ln x \, dx}{1-x^2} = \frac{1}{2} \int_0^1 \frac{\ln x \, dx}{1+x} + \frac{1}{2} \int_0^1 \frac{\ln x \, dx}{1-x}.$$

The values

$$\int_0^1 \frac{\ln x \, dx}{1+x} = -\frac{\pi^2}{12},$$

and

$$\int_0^1 \frac{\ln x \, dx}{1-x} = -\frac{\pi^2}{6},$$

given in 4.231.1 and 4.231.2, respectively, give the result.