

PROOF OF FORMULA 4.231.18

$$\int_0^1 \ln x \frac{1-x^{n+1}}{(1-x)^2} dx = -(n+1) \frac{\pi^2}{6} + \sum_{k=1}^n \frac{n-k+1}{k^2}$$

Start with

$$\int_0^1 \ln x \frac{1-x^{n+1}}{(1-x)^2} dx = \int_0^1 \frac{\ln x}{1-x} \sum_{j=0}^n x^j,$$

and expanding $1/(1-x)$ in series yields

$$\int_0^1 \ln x \frac{1-x^{n+1}}{(1-x)^2} dx = \sum_{j=0}^n \sum_{m=0}^{\infty} \int_0^1 x^{j+m} \ln x dx.$$

The change of variable $u = -\ln x$ gives

$$\int_0^1 \ln x \frac{1-x^{n+1}}{(1-x)^2} dx = - \sum_{j=0}^n \sum_{m=0}^{\infty} \frac{1}{(j+m+1)^2}.$$

The result follows from writing

$$\sum_{m=0}^{\infty} \frac{1}{(j+m+1)^2} = \sum_{m=1}^{\infty} \frac{1}{m^2} - \sum_{m=1}^j \frac{1}{m^2}.$$