

PROOF OF FORMULA 4.231.5

$$\int_0^\infty \frac{\ln x \, dx}{(x+a)^2} = \frac{\ln a}{a}$$

Let $x = at$ to obtain

$$\int_0^\infty \frac{\ln x \, dx}{(x+a)^a} = \frac{\ln a}{a} \int_0^\infty \frac{dt}{(1+t)^2} + \frac{1}{a} \int_0^\infty \frac{\ln t \, dt}{(1+t)^2}.$$

The first integral is 1 and the second one vanishes because the change of variables $t \mapsto 1/t$ shows that

$$\int_0^1 \frac{\ln t \, dt}{(1+t)^2} = - \int_1^\infty \frac{\ln t \, dt}{(1+t)^2}.$$