

PROOF OF FORMULA 4.231.8

$$\int_0^{\infty} \frac{\ln x \, dx}{a^2 + b^2 x^2} = \frac{\pi}{2ab} \ln \frac{a}{b}$$

Formula 4.231.7 states that

$$\int_0^{\infty} \frac{\ln x \, dx}{(a^2 + b^2 x^2)^n} = \frac{\Gamma(n - \frac{1}{2}) \sqrt{\pi}}{4(n-1)! a^{2n-1} b} \left[2 \ln \frac{a}{2b} - \gamma - \psi\left(n - \frac{1}{2}\right) \right].$$

In the special case $n = 1$, this gives

$$\int_0^{\infty} \frac{\ln x \, dx}{a^2 + b^2 x^2} = \frac{\pi}{4ab} \left[2 \ln \left(\frac{a}{2b} \right) - \gamma - \psi\left(\frac{1}{2}\right) \right].$$

The special value

$$\psi\left(\frac{1}{2}\right) = -\gamma - 2 \ln 2$$

gives the result.