PROOF OF FORMULA 4.231.9

$$\int_0^\infty \frac{\ln px\, dx}{q^2+x^2} = \frac{\pi}{2q}\ln pq$$

Let $t = px$ to obtain

$$\int_0^\infty \frac{\ln px\, dx}{q^2+x^2} = p\int_0^\infty \frac{\ln t\, dt}{t^2+(pq)^2}.$$

The result now follows from formula 4.231.8:

$$\int_0^\infty \frac{\ln x\, dx}{a^2+b^2x^2} = \frac{\pi}{2ab}\ln\frac{a}{b}.$$