

PROOF OF FORMULA 4.234.6

$$\int_0^{\infty} \frac{\ln x dx}{(a^2 + b^2 x^2)(1 + x^2)} = \frac{\pi b}{2a(b^2 - a^2)} \ln \frac{a}{b}$$

The partial fraction decomposition

$$\frac{1}{(a^2 + b^2 x^2)(1 + x^2)} = \frac{1}{(a^2 - b^2)(1 + x^2)} - \frac{b^2}{a^2 - b^2} \frac{1}{a^2 + b^2 x^2}.$$

Therefore

$$\int_0^{\infty} \frac{\ln x dx}{(a^2 + b^2 x^2)(1 + x^2)} = \frac{1}{a^2 - b^2} \int_0^{\infty} \frac{\ln x dx}{1 + x^2} - \frac{b^2}{a^2 - b^2} \int_0^{\infty} \frac{\ln x dx}{a^2 + b^2 x^2}.$$

The second integral is given in entry 4.231.8 as

$$\int_0^{\infty} \frac{\ln x dx}{a^2 + b^2 x^2} = \frac{\pi}{2ab} \ln \frac{a}{b}.$$

The special case $a = b = 1$ shows that the first integral vanishes. This gives the result.