PROOF OF FORMULA 4.234.7

$$\int_0^\infty \frac{\ln x \, dx}{(x^2+a^2)(1+b^2x^2)} = \frac{\pi}{2(1-a^2b^2)} \left(\frac{\ln a}{a} + b \ln b \right)$$

The change of variables $t = bx$ gives

$$\int_0^\infty \frac{\ln x \, dx}{(x^2+a^2)(1+b^2x^2)} = \frac{1}{b} \int_0^\infty \frac{\ln t \, dt}{(a^2+t^2/b^2)} - \frac{\ln b}{b} \int_0^\infty \frac{dt}{(t^2/b^2+a^2)(1+t^2)}.$$

The first integral is entry 4.234.6 with b replaced by $1/b$. The second integral can be evaluated as $\pi b/(2a(1+ab))$ by using the partial fracton decomposition

$$\frac{1}{(a^2+t^2/b^2)(1+t^2)} = \frac{b^2}{a^2b^2-1} \frac{1}{t^2+1} - \frac{b^2}{a^2b^2-1} \frac{1}{a^2b^2+t^2}.$$

This gives the result.