PROOF OF FORMULA 4.235.4

$$\int_0^1 \frac{x^{m-1}+x^{n-m-1}}{1-x^n} \ln x \, dx = -\frac{\pi^2}{n^2 \sin^2(\pi m/n)}$$

Start with

$$\int_0^1 \frac{x^{m-1}+x^{n-m-1}}{1-x^n} \ln x \, dx = \int_0^1 \frac{x^{m-1}}{1-x^n} \ln x \, dx + \int_0^1 \frac{x^{n-m-1}}{1-x^n} \ln x \, dx.$$

Entry 4.254.1 gives

$$\int_0^1 \frac{x^{p-1} \ln x}{1-x^q} dx = -\frac{1}{q^2}\psi'\left(\frac{p}{q}\right).$$

Therefore

$$\int_0^1 \frac{x^{m-1}+x^{n-m-1}}{1-x^n} \ln x \, dx = -\frac{1}{n^2}\left[\psi'\left(\frac{m}{n}\right)+\psi'\left(1-\frac{m}{n}\right)\right].$$

The result follows from the relation

$$\psi'(a)+\psi'(1-a) = \frac{\pi^2}{\sin^2 \pi z}.$$