

PROOF OF FORMULA 4.236.1

$$\int_0^1 \left[\frac{1 + (p-1) \ln x}{1-x} + \frac{x \ln x}{(1-x)^2} \right] x^{p-1} dx = -1 + \psi'(p)$$

The formula

$$\frac{d}{dx} \left(\frac{x \ln x}{1-x} \right) = \frac{1}{1-x} + \frac{\ln x}{1-x} + \frac{x \ln x}{(1-x)^2}$$

gives

$$\int_0^1 \left[\frac{1 + (p-1) \ln x}{1-x} + \frac{x \ln x}{(1-x)^2} \right] x^{p-1} dx = \int_0^1 \left[x^{p-1} \frac{d}{dx} \left(\frac{x \ln x}{1-x} \right) + (p-2) \frac{x^{p-1} \ln x}{1-x} \right] dx.$$

Now use

$$\frac{d}{dx} \left[x^{p-1} \left(\frac{x \ln x}{1-x} \right) \right] = x^{p-1} \frac{d}{dx} \left[\frac{x \ln x}{1-x} \right] + (p-1)x^{p-2} \frac{x \ln x}{1-x}$$

to obtain

$$\int_0^1 \left[\frac{1 + (p-1) \ln x}{1-x} + \frac{x \ln x}{(1-x)^2} \right] x^{p-1} dx = \int_0^1 \frac{d}{dx} \left[\frac{x^p \ln x}{1-x} \right] dx - \int_0^1 \frac{x^{p-1} \ln x}{1-x} dx.$$

The first integral is -1 and the second one appears from differentiating the identity

$$\psi(p) = - \int_0^1 \left(\frac{x^{p-1}}{1-x} + \frac{1}{\ln x} \right) dx.$$

This is the result.