PROOF OF FORMULA 4.241.6

$$\int_0^{1/\sqrt{2}} \frac{\ln x\, dx}{\sqrt{1-x^2}} = -\frac{\pi}{4}\ln 2 - \frac{G}{2}$$

Let $x = \sin t$ to obtain

$$\int_0^{1/\sqrt{2}} \frac{\ln x\, dx}{\sqrt{1-x^2}} = \int_0^{\pi/4} \ln\sin t\, dt.$$

This integral is evaluated in 4.224.2.