PROOF OF FORMULA 4.241.7

$$\int_0^1 \frac{\ln x\, dx}{\sqrt{1-x^2}} = -\frac{\pi}{2}\ln 2$$

Let $x = \sin t$ to obtain

$$\int_0^1 \frac{\ln x\, dx}{\sqrt{1-x^2}} = \int_0^{\pi/2} \ln(\sin t)\, dt.$$

This integral is evaluated as $-\frac{\pi}{2}\ln 2$ in 4.224.3.