

PROOF OF FORMULA 4.241.9

$$\int_0^1 \sqrt{1-x^2} \ln x \, dx = -\frac{\pi}{8}(1+2\ln 2)$$

Let $t = -\ln x$ to obtain

$$\int_0^1 \sqrt{1-x^2} \ln x \, dx = -\int_0^\infty t e^{-t} \sqrt{1-e^{-2t}} \, dt.$$

This last integral is evaluated in 3.451.2.