

**PROOF OF FORMULA 4.244.1**

$$\int_0^1 \frac{\ln x \, dx}{\sqrt[3]{x(1-x^2)^2}} = -\frac{1}{8} \left[ \Gamma\left(\frac{1}{3}\right) \right]^3$$

In the proof of formula 4.253.1 it was shown that

$$\int_0^1 t^{a-1}(1-t)^{b-1} \ln t \, dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} [\psi(a) - \psi(a+b)].$$

The change of variable  $t = x^2$  gives

$$\int_0^1 \frac{\ln x \, dx}{\sqrt[3]{x(1-x^2)^2}} = \frac{1}{4} \int_0^1 t^{-2/3}(1-t)^{-2/3} \ln t \, dt.$$

It follows that

$$\int_0^1 \frac{\ln x \, dx}{\sqrt{x(1-x^2)}} = \frac{1}{4} \frac{\Gamma(1/3)\Gamma(1/3)}{\Gamma(2/3)} [\psi(1/3) - \psi(2/3)].$$

The identities

$$\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin \pi x} \text{ and } \psi(x) - \psi(1-x) = -\pi \cot(\pi x),$$

is now used to produce the result.