

PROOF OF FORMULA 4.247.1

$$\int_0^1 \frac{\ln x \, dx}{\sqrt[n]{1-x^{2n}}} = -\frac{\pi}{8n^2 \sin(\pi/2n)} B\left(\frac{1}{2n}, \frac{1}{2n}\right).$$

The change of variable $t = x^{2n}$ gives

$$\int_0^1 \frac{\ln x \, dx}{\sqrt[n]{1-x^{2n}}} = \frac{1}{4n^2} \int_0^1 t^{1/2n-1} (1-t)^{-1/n} \ln t \, dt.$$

In the proof of formula 4.253.1 it was shown that

$$\int_0^1 t^{a-1} (1-t)^{b-1} \ln t \, dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} [\psi(a) - \psi(a+b)].$$

Therefore

$$\int_0^1 \frac{\ln x \, dx}{\sqrt[n]{1-x^{2n}}} = \frac{\Gamma(1/2n)\Gamma(1-1/n)}{4n^2\Gamma(1-1/2n)} \left[\psi\left(\frac{1}{2n}\right) - \psi\left(1 - \frac{1}{2n}\right)\right].$$

The relations

$$\begin{aligned} \Gamma\left(1 - \frac{1}{n}\right) \Gamma\left(\frac{1}{n}\right) &= \pi / \sin(\pi/n) \\ \psi\left(\frac{1}{2n}\right) - \psi\left(1 - \frac{1}{2n}\right) &= -\pi \cot(\pi/2n) \end{aligned}$$

give the result.