

PROOF OF FORMULA 4.251.1

$$\int_0^\infty \frac{x^{\mu-1} \ln x}{b+x} dx = \frac{\pi b^{\mu-1}}{\sin \pi \mu} (\ln b - \pi \cot \pi \mu)$$

Let $x = bt$ to obtain

$$\int_0^\infty \frac{x^{\mu-1} \ln x}{b+x} dx = b^{\mu-1} \ln b \int_0^\infty \frac{t^{\mu-1} dt}{1+t} + b^{\mu-1} \int_0^\infty \frac{t^{\mu-1} \ln t}{1+t} dt.$$

The integral representation for the beta function

$$B(\alpha, \beta) = \int_0^\infty \frac{t^{\alpha-1} dt}{(1+t)^{\alpha+\beta}},$$

gives

$$\int_0^\infty \frac{t^{\mu-1} dt}{1+t} = B(\mu, 1-\mu) = \Gamma(\mu)\Gamma(1-\mu) = \frac{\pi}{\sin \pi \mu}.$$

Differentiating with respect to μ yields

$$\int_0^\infty \frac{t^{\mu-1} \ln t}{1+t} dt = -\pi^2 \frac{\cos \pi \mu}{\sin^2 \pi \mu}.$$