PROOF OF FORMULA 4.251.5

$$\int_0^1 \frac{x^{2n}\ln x\,dx}{1+x} = -\frac{\pi^2}{12} + \sum_{j=1}^{2n} \frac{(-1)^{j-1}}{j^2}$$

Let $t = -\ln x$ to obtain

$$\int_0^1 \frac{x^{2n}\ln x\,dx}{1+x} = -\int_0^\infty \frac{te^{-2nt}\,dt}{1+e^t}.$$

This integral is evaluated in 3.411.12.